Reply to Comment on "Can gravity distinguish between Dirac and Majorana neutrinos?"

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(Dated: February 15, 2009)

This comment is in response to Nieves and Pal [1] who dispute our claim [2] that a classical gravitational field may possibly distinguish between Dirac and Majorana neutrinos, described in terms of Gaussian wave packets propagating in a Lense-Thirring background, where the distinction is manifested in spin-gravity corrections to the neutrino oscillation length. They contend that our model for Majorana neutrinos is incorrect and that any conclusions relating to this neutrino type are not reliable. Furthermore, they suggest that any distinction between the two neutrino types will be suppressed by factors of m/E where m is the neutrino mass and E is its mean energy of propagation, and claim that this distinction is unobservable when $m/E \ll 1$. We beg to disagree.

As noted above, our model for the Dirac and Majorana neutrinos is motivated by a wave packet approach in *quantum mechanics*, as opposed to a plane-wave expansion in *quantum field theory*. This is an essential detail which Nieves and Pal fail to acknowledge. In addition, the gravitational field [2] is incorporated in terms of a gravitational phase $\Phi_{\rm G}$, giving rise to an interaction Hamiltonian $H_{\Phi_{\rm G}}$ with spin-dependent features, to be evaluated in terms of time-independent perturbation theory. These two details are important for framing the context underpinning our reply. Regarding the technical concerns, we agree that the Majorana condition they note in their eq. (1) is certainly true for a *fermion field operator*. However, we again emphasize that our perspective is *quantum mechanical*, so our treatment of the Majorana condition must be described in terms of *wave functions*. Adopting their notation, our approach is to identify [3, 4] u_R with $(u_L)^c = i\,\sigma^2\,u_L^*$ and also separately identify u_L with $(u_R)^c = -i\,\sigma^2\,u_R^*$ to obtain [5]

$$W_1 \;=\; u_L + (u_L)^c, \qquad W_2 \;=\; u_R - (u_R)^c. \tag{1}$$

Unlike what Nieves and Pal claim, it indeed follows [3] that $W_{1,2}$ is a solution of the free particle equation $\not{k}\,W_{1,2} = \pm\,m\,W_{1,2}$ [5] for wave functions. However, a more precise implementation of (1) with our notation leads to a Majorana wave packet model with the form

$$|\psi_{1(2)}\rangle^{\rm Maj.} \;=\; \frac{1}{(2\pi)^{3/2}} \int {\rm d}^3k\,\xi(\boldsymbol{k})\,|W_{1(2)}(\boldsymbol{k})\rangle^{\rm Maj.}, \tag{2}$$

where

$$|W_1(\boldsymbol{k})\rangle^{\rm Maj.} \;=\; e^{-ik\cdot x}|\nu_L\rangle + e^{ik\cdot x}|\nu_L^c\rangle, \tag{3}$$
$$|W_2(\boldsymbol{k})\rangle^{\rm Maj.} \;=\; e^{-ik\cdot x}|\nu_R\rangle - e^{ik\cdot x}|\nu_R^c\rangle, \tag{4}$$

and $\xi(\boldsymbol{k})$ is the Gaussian function [2]. Clearly, it follows from (2) that $|\psi^c_{1(2)}\rangle^{\rm Maj.} = \pm\,|\psi_{1(2)}\rangle^{\rm Maj.}$. This leads to a modification of our eq. (13) for the Majorana matrix element [2], which is

$$\begin{aligned}\langle\psi_{1(2)}(\boldsymbol{r})|H_{\Phi_{\rm G}}|\psi_{1(2)}(\boldsymbol{r})\rangle^{\rm Maj.} \;&=\; \langle\psi(\boldsymbol{r})|H_{\Phi_{\rm G}}|\psi(\boldsymbol{r})\rangle^{\rm Dirac} \\ &\pm (\hbar\,k_0)\sin\theta\,\sin\varphi\left[\frac{M}{r}\langle\pm|\boldsymbol{\sigma}|\mp\rangle^{\hat{y}}\left[C_{0\hat{y}} + C_{1\hat{y}}\,\bar{m} + C_{2\hat{y}}\,\bar{m}^2\right]\right. \\ &+ \left.\frac{M\Omega R^2}{r^2}\langle\pm|\boldsymbol{\sigma}|\mp\rangle^{\hat{x}}\left[D_{0\hat{x}} + D_{1\hat{x}}\,\bar{m} + D_{2\hat{x}}\,\bar{m}^2\right]\right].\end{aligned} \tag{5}$$

Our plots [2], as applied to the SN 1987A data, are *completely unaffected* by the adjustments because the corrections only alter the contributions coupled to $M\Omega R^2/r^2$, which are all exponentially damped compared to the M/r contributions. As for the m/E suppression issue raised by Nieves and Pal, this is of no relevance because all such terms are *automatically excluded* within the construction of $|W_{1,2}(\boldsymbol{k})\rangle^{\rm Maj.}$, so everything presented in [2] is of leading order. As shown in (5), and also present in our eq. (13) of [2], Dirac and Majorana neutrinos behave differently under the gravitational perturbation $\Phi_{\rm G}$ by way of the spin-flip terms $\langle\pm|\boldsymbol{\sigma}|\mp\rangle^{\hat{x},\hat{y}}$. A spin-flip changes a Majorana neutrino into an antineutrino and behaves like a charge conjugation operation.

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[1] J.F. Nieves and P.B. Pal, arXiv:gr-qc/0610098 (2006); *Phys. Rev. Lett.* (in press).
[2] D. Singh, N. Mobed, and G. Papini, *Phys. Rev. Lett.* **97**, 041101 (2006).
[3] I.J.R. Aitchison and A.J.G. Hey, *Gauge Theories in Particle Physics - Volume II: QCD and the Electroweak Theory, 3rd Edition* (IOP Publishing 2004).
[4] M. Fukugita and T. Yanagida, *Physics of Neutrinos and Applications to Astrophysics*, (Springer-Verlag Press 2003).
[5] Please note the typographical error present in their eq. (4) and related expressions.